Math 20550 - Summer 2016
Fundamental Theorems Worksheet
July 18, 2016

All of the following questions use one of the following: (1) fundamental theorem of line integrals, (2) Green's Theorem, (3) Stokes' Theorem, or (4) the Divergence Theorem. Start by figuring out which theorem applies to each problem and how you would use the theorem to compute the integral, then worry about actually computing the integrals. (It may be possible to apply more than one of the four theorems to a given problem.)

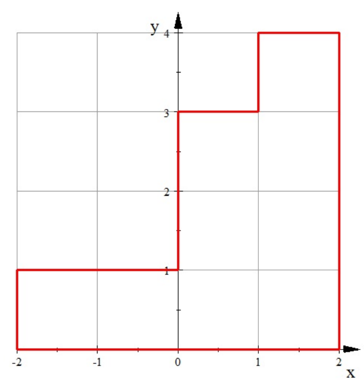
Question 1. *Let S be the surface defined as $z = 4 - 4x^2 - y^2$ with $z \geq 0$ and oriented upward. Let $\mathbf{F} = \langle x - y, x + y, ze^{xy} \rangle$. Compute* $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$.

Question 2. *Evaluate*

$$\int_C (y^3 + \cos x)dx + (\sin y + z^2)dy + x\,dz$$

where C is the closed curve parametrized by $\mathbf{r}(t) = \langle \cos t, \sin t, \sin 2t \rangle$. (Hint: the curve C lies on the surface $z = 2xy$.)

Question 3. *Evaluate* $\oint_C (x^4y^5 - 2y)dx + (3x + x^5y^4)dy$ *where C is the curve below*



Question 4. *Calculate the work done by the vector field $\mathbf{F} = \langle 4y - 3x, x - 4y \rangle$ on a particle as the particle moves counterclockwise once around the ellipse $x^2 + 4y^2 = 4$.*

Question 5. *Compute* $\int_C \mathbf{F} \cdot d\mathbf{r}$ *where*

$$\mathbf{F} = \big\langle (2x + z)\cos(x^2 + xz), -(z + 1)\sin(y + yz), x\cos(x^2 + xz) - y\sin(y + yz) \big\rangle$$

and C is parametrized by $\mathbf{r}(t) = \left\langle t^3, t^2, \pi t - \sin \frac{\pi t}{2} \right\rangle$, $0 \leq t \leq 1$.

Question 6. *Compute the flux of $\mathbf{F} = xyz\mathbf{j}$ across the cylinder with lateral surface $x^2 + y^2 = 4$, top $z = 5$, and bottom $z = 0$, and outward orientation.*

Question 7. *Compute* $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$, *where $\mathbf{F} = \langle 4xy + z^2, 2x^2 + 6yz, 2xz \rangle$ and S is the closed surface in the first octant bounded by the coordinates planes, $x = 4$, and $z = 9 - y^2$, given outward orientation.*

Question 8. *Compute* $\oint_C (y - x)dx + (2x - y)dy$ *where C is the boundary of the region lying inside the rectangle bounded by $x = -5$, $x = 5$, $y = -3$, and $y = 3$, and outside the square bounded by $x = -1$, $x = 1$, $y = -1$, and $y = 1$.*

Question 9. *Evaluate* $\int_C (x^2+y^2)dx + 2xy\,dy$ *where* C *is parametrized by* $\mathbf{r}(t) = t^3\mathbf{i} + t^2\mathbf{j}$, $0 \le t \le 2$.

Question 10. *Find the flux of* $\mathbf{F} = x\mathbf{i} + y^2\mathbf{j} - 2yz\mathbf{k}$ *across the surface which is the boundary of the region bounded by* $z = \sqrt{36 - x^2 - y^2}$ *and* $z = 0$, *with outward orientation.*

Question 11. *Let* S *be the surface described by* $z = e^{1-x^2-y^2}$, $z \ge 1$, *oriented with upward normals. Let* $\mathbf{F} = \langle x, y, 2-2z \rangle$ *and compute the flux of* $\mathbf{F}$ *across* S.

Question 12. *Compute* $\int_C \nabla f \cdot d\mathbf{r}$ *where a contour plot of* f *is provided below and* C *is the curve drawn which starts at* A *and ends at* B.